|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | s | |  | | --- | | p→s | | |  | | --- | | q∧(p→s) | | |  | | --- | | q∧s | | p, q ∧ (p → s) |= q ∧ s |
| T | T | T | T | T | T | T |
| T | T | F | T | F | F | F |
| T | F | T | T | F | F | F |
| T | F | F | F | F | F | F |
| F | T | T | T | T | T | F |
| F | T | F | T | T | F | F |
| F | F | T | T | F | F | F |
| F | F | F | T | F | F | F |

Logic Assessment

1.

p, q ∧ (p → s) |= q ∧ s construct a truth table to evaluate whether p and q∧(p→s) logically entail (∣=) q∧s.

From the truth table we observe that p and q∧(p→s) do not logically entail (∣=) q∧s.

2.

R = in shape

Q = healthy

P = unhealthy

R → Q |= ¬Q → (P ∨ ¬R)

If a person is in shape (R) then they are healthy (Q) and this logically entails that if a person is not healthy (¬Q) then they are unhealthy or not in shape (P ∨ ¬R) .

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| P | Q | R | ¬Q | R → Q | ¬Q → (P ∨ ¬R) | R → Q |= ¬Q → (P ∨ ¬R) |
| T | T | T | F | T | T | T |
| T | T | F | F | T | T | T |
| T | F | T | T | F | T | F |
| T | F | F | T | F | F | F |
| F | T | T | F | T | T | T |
| F | T | F | F | T | T | T |
| F | F | T | T | T | T | T |
| F | F | F | T | T | T | T |

Thus, the truth table shows that whenever R → Q is true, ¬Q → (P ∨ ¬R) must be true, and this demonstrates that the implication R → Q |= ¬Q → (P ∨ ¬R) holds.

3. Prove the following, using the semantic equivalences given in Section 3 of the Logic Notes. Prove step by step, using one equivalence at a time.

(a) ¬p → ¬p ≡ true  [8 marks]

LHS

¬p → ¬p ≡ ¬(¬p) ∨ p (implication)

¬(¬p) ∨ ¬p ≡ p ∨ ¬p (double negation)

p ∨ ¬p ≡ true (negation)

(b) (q → r) ∧ (p ∨ r) ≡ (p → q) → r  [16 marks]

LHS

(q → r) ∧ (p ∨ r)  ≡  ((¬q)  ∨  r)   ∧  (p ∨ r)  (implication)

((¬q)  ∨  r)   ∧  (p ∨ r)  ≡ r  ∨ ( ¬q  ∧ p ) (distributivity)

r  ∨ ( ¬q  ∧ p )  ≡ r ∨ ¬(¬p ∧ q) (de Morgan)

r ∨ ¬(¬p ∧ q) ≡ r ∨ ¬(p → q) (implication)

r ∨ ¬(p → q) ≡ ¬(p → q) ∨ r (communicativity)

¬(p → q) ∨ r ≡ (p → q) → r (implication)